ASSIGNMENT OF SPECTRUM DEMANDS BY MERITS VIA ANALYTIC HIERARCHY PROCESS AND INTEGER PROGRAMMING

Razamin Ramli[1], Faathirah Jamaluddin[2], Engku Muhammad Nazri Engku Abu Bakar[3], Mohamad Yusoff Alias[4], Nor Idayu Mahat[5] and Mohd Zaini Abdul Karim[6]

[1,2,3,5]*School of Quantitative Sciences, UUM College of Arts and Sciences*
[6] *School of Economics, Finance and Islamic Banking*
UUM College of Business,
Universiti Utara Malaysia
06010 Sintok, Kedah, Malaysia

[4]*Faculty of Engineering, Multimedia University*

Corresponding author: razamin@uum.edu.my[1]

ABSTRACT

Demand for wireless services is ever increasing due to wireless technology advances, which directly relates to the increasing demand for spectrum bandwidths. Due to the high demand, thus spectrum has become scarce. As a result, spectrum bandwidths have become important and need to be efficiently assigned to potential and demanding service providers. Thus, this paper proposes an integrated approach to solve a spectrum assignment problem. The integrated model is developed to efficiently determine the optimal spectrum assignment for the purpose of satisfying the identified relevant requirements and constraints or criteria. The development of the model has also taken into consideration the situation when the spectrum regulator wishes to choose the best provider or licensee based on merits. For that purpose, suitable weights based on qualitative criteria were obtained via Analytic Hierarchy Process and subsequently used as coefficients in the respective Integer Programming formulation. The integrated optimization model is able to tackle a multi-criteria spectrum problem and then solve the assignment problem by way of maximizing the total efficiency. The results exhibits a more efficient alternative as compared to the existing qualitative approach, as it is able to combine subjective judgments

computation with a mathematically formulated approach to produce a systematic and consistent results.

INTRODUCTION

Resource allocation involves assigning the available resources in an economic way and at the same time fulfilling certain organizational constraints (Gardent & Reeves, 2009). The importance of efficient resource allocation has been proven in several previous studies, such as the emergency resource allocation in relation to missile allocation (Matlin, 1970), resource allocation for emergency response after earthquake disaster (Fiedrich, Gehbauer, & Rickers, 2000), donors' allocation (Kriesche, Schold, Gaston, Wadstrom, & Kaplan, 2005), allocation of multiple emergency service resources (Huang, Fan, & Cheu, 2006), shelf allocation (Bultez & Naert, 1988), warehouse allocation (Lashine, Fattouh, & Issa, 2006), airport slot allocation (Madas & Zografos, 2010), and agriculture land allocation (Gilbert, Holmes & Rosenthal, 1985). Other related works in telecommunication dealt with resource allocation involving a number of processes in a network of processors (Sofianopoulou, 1992), strategies for real-time services (Babich & Deotto, 2002), channel allocation (Chang & Kim, 1997; Kalvenes, Kennington, & Olinick, 2005; Krishna, Misra, Obaidat, & Saritha, 2009), and spectrum demand allocation (Engku, Razamin Ramli, Ang Chooi Leng, Nor Idayu Mahat, Mohd Zaini Abdul Karim & Mohd Yusof Alias, 2009) or more commonly called spectrum assignment.

Since telecommunication industry is a fast-moving industry that may generate surprisingly large revenues through the management of spectrum as the main resource, therefore it is deemed important that the available spectrum should be managed efficiently. As the demand for wireless service is high, the demand for spectrum bandwidth increases and thus, spectrum has become scarce. Spectrum refers to a collection of various types of electromagnetic radiations of different wavelengths (Fette, 2006). The demand for spectrum is ever increasing with the advancement of a variety of mobile radio communication services, the introduction of new mobile radio communication services one after another (Sekiguchi, Ishikawa, Koyama, & Sawada, 1986), and also the upgrading of future generations of wireless technologies (Kulkarni & Zekavat, 2006). This adds to the problem of spectrum scarcity as further suggested by Zhao, Peng, Zheng and Shang (2009).

In addition, the scarcity of spectrum may also be due to today's telecommunication trend, where current available telecommunication service providers are demanding for much spectrum simultaneously. At the same time, when a new service provider enters the market, a lack of spectrum will occur as demands may outweigh the supply due to increased competition. As a result, spectrum bandwidths need to be efficiently allocated or assigned to the potential and demanding service providers, a task which is the main focus of this paper. The main efficiency issue in spectrum management is the assignment process. Spectrum assignment involves designating bands of spectrum to specific types of services or classes of users, such as designating certain bands for commercial use and others for government use.

Subsequently, a more efficient usage of the available spectrum is needed in the course of managing the services given by providers who in turn are able to offer better wireless communication services to the society. Hence, the efficient utilization of the scarcely available radio spectrum becomes a fundamental problem as agreed by Sekiguchi, Ishikawa, Koyama and Sawada (1986), Tzifa, Demestichas, Louta, Tsouka, Theologuo and Anagnostou (1999), and Pattavina, Quadri and Trecordi (1999). Therefore, what is the appropriate approach in planning and assigning the available spectrum bandwidths so as to maximize the efficient use of spectrum resources?

As such, this paper proposes an efficient approach to assign spectrum bandwidths to assist the regulating body (or also known as the regulator) in assigning the spectrum in a fair manner. The approach would be able to help the regulating body in planning and assigning the spectrum in the most suitable and beneficial ways, for example, by determining the size of spectrum that needs to be assigned. The paper is as follows: The next section of the paper continues with the issues on current spectrum assignment approaches, followed by discussion on some further related work, the proposed integrated model, and results and analysis. The final section concludes the paper with recommendations for some future work.

CURRENT SPECTRUM ALLOCATION APPROACH

Several approaches of allocating or awarding the spectrum licenses to service providers such as the auction, beauty contest, and lottery have been illustrated from the current related literature (Baijal, 2004; Engku et al., 2009). An auction is a multi-criteria decision making approach used by a regulating body to select the best service providers among the competing ones. It is a transparent selection exercise which involves multiple criteria normally set by the spectrum regulator. On the other hand, a beauty contest is another approach

for evaluating service providers before the spectrum assignment is made which involves proposed outlines and criteria to be followed in the selection process of the service provider. Beauty contest is also called the administrative process or bureaucratic method. In a similar spectrum assignment, Andersson, Hulten, & Valiente (2005) conducted a study using the beauty contest approach in the 3G licensing process in Sweden. However, auction is better than beauty contest because it offers a more market-oriented, objective and transparent approach for awarding spectrum licenses (Andersson et al., 2005; Binmore & Klemperer, 2001; Klemperer, 2000; McMilan, 1995).

Similarly, Cramtom (2002) argued that beauty contest is extremely slow, wasteful and lacks transparency. He was most interested in lottery which is a qualitative approach involving some kind of element of fate or luck. Cramtom (2002) also agreed that the lottery approach could be successful in assigning licenses quickly since the prospect of a windfall gain may attract a large number of applicants. Specifically, in a lottery, licensees are selected at random from among all competing spectrum applicants (Zanakis, Solomon, Wishart, & Dublish, 1998). Lotteries can decrease some aspects of the administrative burden entailed in comparative hearings, such as legal expenses, but may create a different kind of administrative burden by encouraging more applications to be filed. In addition, lotteries do not assign spectrum to those who value it most highly, except by chance. Hence, this leads to significant transaction costs, and again generate no revenues, unless fees are attached to the license assigned by lottery, or an entry fee to participate in the lottery is charged.

All of the approaches described above use some kind of determining criteria to assign the spectrum. Certain criteria were taken into consideration but were only based on subjective judgments. However, based on the literature and also in-depth discussions with experts in the regulating body, the important criteria for consideration in spectrum allocation are such as economic efficiency, promotion of competition, fairness, revenue maximization, fulfillment of specific requirements (e.g. geographic coverage, obligations relating to speed, cost of rollout and quality of service), encouraging innovation and investment in the telecommunication sector (Engku et al., 2009), and encouraging green technology (Fathirah Jamaluddin, 2011). It is certainly a difficult task to perform when trying to make a decision on which best service provider is to be awarded the spectrum volumes using any of the qualitative approaches, while considering all the above criteria simultaneously. The effort is unfortunately prone to biasness and unfairness. Hence, an efficient approach or technique must be given a deep thought and consideration as an alternative to solve the assignment problem.

FURTHER RELATED WORK

Some similar works on allocating or assigning scarce resources like land were referred to earlier as there is no previous work on spectrum assignment using any efficient approach such as the mathematical programming (MP) techniques, prior to the study by Razamin Ramli et al. (2011). For example, the work by Gilbert, Holmes & Rosenthal (1985) dealt with the allocation of land areas for development using a multi-objective integer programming (MIP) model, which is an alternative MP technique. The criteria that were considered in this allocation problem were cost, proximity to desirable and undesirable land features, and the shape of area, which were successfully applied for a development area in Tennessee.

When there is a need to quantify subjective judgments which involve multi-criteria decision making, the Analytic Hierarchy Process (AHP) is one alternative and effective method to employ (Malczewski, Moreno-Sanchez, Bojo´rquez-Tapia, & Ongay-Delhumeau, 1997). In the work by Malczewski et al. (1997), another variant of MP that is, the integer programming (IP) was combined with the AHP to aid land use planning. The AHP was initially applied in order to obtain relative importance in terms of weights for alternative interest groups, a concept similar to the creation of the level of importance as done by Maznah Mat Kasim, Haslinda and Mahmoud Salem (2011). The weights were then utilized as weighting factors in the objective function of the IP model, which aimed at maximizing the consensus among the interest groups.

In a similar approach, AHP and Linear Programming (LP), other variants of MP were integrated to solve an allocation problem for alternative material handling devices. The criteria involved were cost, benefit, and compatibility of each device with respect to each manufacturing cell, whereby the objective was to select a set of devices with the maximum weights (Braglia, Gabbrielli, & Miconi, 2001).

Recently, a spectrum assignment problem was investigated (Razamin Ramli et al., 2011) based on the situation faced by a regulating body, which is responsible for distributing spectrum bandwidths to a number of wireless service providers. It was understood that an efficient technique such as the mathematical modeling approach has never been applied in this case problem. Existing approaches that were being implemented when assigning the required bandwidths are of qualitative types, such as beauty contest (Binmore & Klemperer, 2001) and auction (Morris, 2005). However, in this paper we introduce another variant of the spectrum assignment model based on merits,

which actually differs from the model by Razamin Ramli et al. (2011) as the latter was based on request by service providers. The criteria involved in the model are still the same as those in that model.
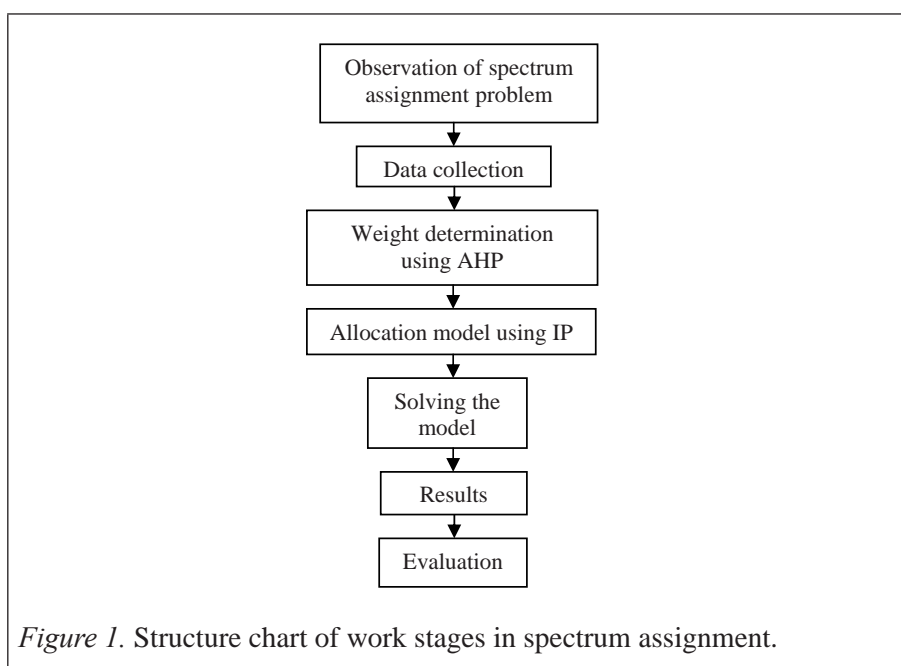
THE PROPOSED INTEGRATED MODEL

The proposed approach follows the track of an integration of AHP and IP, which is in line with other similar studies, as discussed earlier, that is an attempt to capture the advantages of both techniques. In other words, it can be seen that the integration approach exploits the strengths of both techniques (Jain & Kumar, 2007). Specifically, by integrating AHP and IP techniques, we are able to combine subjective judgments with a mathematically-formulated approach to produce a systematic and efficient result (Malczewski et al., 1997; Razamin Ramli et al., 2011). As a general guide, one of the principles promoted by Davis (1991) which could be beneficial to consider is hybridize or integrate where possible. The principle suggests that by doing so, it can incorporate the positive features of the current algorithm or technique into the hybrid or integrated approach.

In the current assignment process, certain criteria were considered throughout the process but in a subjective manner until the final decision was made. When exercising the process of spectrum assignment, the regulating body may have different preferences or judgments for different licensees in relation to each of these criteria. Hence, the integrated approach provides a potential avenue to solve the problem of assigning spectrum to service providers by considering all related criteria so that the assignment can be more efficiently made. The proposed approach thus integrates quantifiable qualitative judgments through AHP computation into the IP.

In order to show the feasibility of the proposed integrated approach, a case problem was chosen but the criteria considered in this problem were not quite clear. Thus, based on the case and enhancement from the literature, we conclude that seven criteria and four service providers are to be considered in the proposed decision making process of assigning spectrum. A service provider or licensee is the company which provides the telecommunication service. For reasons of confidentiality, we avoid the usage of original names but instead identified the four licensees as A, B, C, and D.

These criteria are economic efficiency, promotion of competition, fairness, revenue maximization, quality of service, encouragement for innovation and investment, and encouragement for green technology. For details of

each criterion, please refer to Fathirah Jamaluddin (2011). Presently, the assignment of spectrum bandwidths involves various ranges or sizes of the bandwidth. however, for the purpose of illustrating the proposed approach, only the spectrum in Global System for Mobile Communication (GSM) in 2G is considered. All of these licensees provide 2G telecommunication, which can be categorized into two types. Type I is the first type, which is for bandwidth in the range of 800-900MHz. The second type is called Type II with the bandwidth in the range of 1800-1900MHz. The model can be extended to any similar situation in other categories if needed. All data regarding the assignment of spectrum was obtained from the regulating body and is very much similar to the real situation where possible. The proposed integrated model is constructed based on the following procedures as illustrated in Figure 1. However, different but similar integrated models need to be constructed for each type of the bandwidth range. Therefore, only the integrated model for Type I is illustrated for discussion in this paper as it is similar.

Figure 1. Structure chart of work stages in spectrum assignment.

The proposed integrated model allows the AHP component to be enclosed in the IP model. The function of AHP is to obtain the weights for each criterion and licensee accordingly, which are then used as the coefficients in the IP formulations.

The integer programming model

The modeling components are described below and these include definition of variables, parameters, objective function, and constraints function. The IP model is constructed such that the spectrum to be assigned is based on merits of the licensees.

x_{ij} = the number of packet from type i was assigned to licensee j

i = type of bandwidth in 2G, $i = 1, \dots, I;$

j = type of licensee, $j = 1, 2, \dots, J;$

k = type of criteria, $k = 1, 2, \dots, K;$

p = number of available packets, where each packet is 2.5 MHz in width;

α_k = weight for criteria $k;$

C_{ijk} = weight for licensee j assigned to bandwidth of type i with respect to criteria $k;$

R_{ij} = required number of packets for bandwidth type i requested by licensee $j;$

E_{ij} = expected number of packets for bandwidth type i requested by licensee $j;$

M = a large number

$$Y_{ij} = \begin{cases} 1 & \text{If required demands, } R_{ij} \text{ is fulfilled} \\ 0 & \text{otherwise} \end{cases}$$

The objective function (1) maximizes Z, the total efficiency that are obtained based on each of the specific criteria, i.e. economic efficiency, promotion of competition, fairness, revenue maximization, quality of service, encouragement for innovation and investment, and encouragement for green technology.

$$\text{Maximize } Z = \sum_{i=1}^{I} \sum_{j=K}^{J} \sum_{=1}^{K} \alpha_k C_{ij} x_{ij} \tag{1}$$

Subject to

$$\sum_{j=1}^{J} x_{ij} \leq p, \ \forall i, \ i = 1, 2, \dots, I; \tag{2}$$

$$x_{ij} \leq MY_{ij} ; \tag{3}$$

$$x_{ij} \geq R_{ij} - M(1-Y_{ij}) ; \tag{4}$$

$$x_{ij} \leq E_{ij} . \tag{5}$$

$$x_{ij} \geq 0 \text{ for each } i, j. \tag{6}$$

Constraint (2) enforces that each type of spectrum cannot exceed the available packets. Constraints (3) and (4) state that x_{ij} must be either 0 or greater than the required demand. Constraint (5) refers to the expected demand constraint which means that, the number of packet x_{ij} must be less than or equal to the expected demand. Both constraints (4) and (5) are demand constraints. The required volume is the minimum spectrum requirement that is needed for the licensee to be operable. The expected volume could guarantee the licensee to operate at the utmost best but may not necessarily be offered to the licensee. Finally, constraint (6) ensures the non-negativity compliance.

The AHP component

Developing a hierarchy structure, which consists of three main levels, i.e., goal-setting, selection of criteria followed by making decision on alternatives is the first step in AHP. Please refer to Taylor (2010) for details of the technique. In order to get a clear picture of the problem, the decision on alternatives and selection of criteria should be clearly determined. Two sets of weight which are weights for criteria and weights for licensees are needed for this case problem.

Weight is the relative importance granted to elements of criteria and licensees based on the judgments of the decision makers, who are the members of the regulating body. The numerical values are obtained based on the AHP scale of 1 – 9 through a series of pair-wise comparisons. After that, the process continues to the synthesization stage and consistency test. These weights are then used as coefficients in the objective function (1).

RESULTS AND ANALYSIS

A series of hierarchical processes of the AHP were computed to obtain weights or relative importance which is the initial solution (in terms of criteria rating) to the integrated model for bandwidth of Type I. In order for the process to take place, prior ratings were generated. The values shown in Table 1 are

the efficiency weights or relative importance of each of the identified seven criteria. Based on decision makers' judgments, licensee A is the best in terms of economic efficiency, while licensee B is the best in terms of promotion of competition, fairness, revenue maximization and also encouragement for green technology. In addition, licensee D is the best in terms of quality of service and encouragement for innovation and investment.

Table 1

Weights or Relative Importance of Each Criterion

Criterion	Weight
Economic efficiency	0.3504
Promotion of competition	0.0462
Fairness	0.1590
Revenue maximization	0.0318
Quality of service	0.0696
Encouragement for innovation and investment	0.1056
Encouragement for green technology	0.2375

Through AHP also, we could obtain the coefficient values or efficiency weights for each licensee in relation to each criterion, which are presented in Table 2. For example, in terms of encouragement for green technology, licensee B scores first, licensee A scores second, licensee D scores third, and licensee C scores last. The description continues similarly with the rest of the criteria.

Table 2

Overall Weights for Each Licensee Based on Criteria in Type I Bandwidth

Criteria	A	B	C	D
Economic efficiency	0.49	0.13	0.31	0.08
Promotion of competition	0.08	0.54	0.14	0.23
Fairness	0.24	0.55	0.16	0.05
Revenue maximization	0.28	0.47	0.17	0.07
Quality of service	0.26	0.07	0.11	0.56
Encouragement for innovation and investment	0.28	0.10	0.16	0.47
Encouragement for green technology	0.28	0.47	0.10	0.16

Subsequently, in implementing the proposed integrated model, we tested a case situation where $I = 1$ (i.e. for Type I only), $J = 4$, $K = 7$, and $p = 40$ as observed. The values for parameters R and E for all i and j were obtained and are depicted in Table 3.

Table 3

Supply Volume, Required and Expected Demands of Spectrum for Type I Bandwidth

Licensee	Type I	
	Required demand, R	Expected demand, E
A	14	15
B	10	14
C	7	14
D	8	13
Supply volume	40	

After all parameters have been inserted, the integrated IP model was run using the Lingo 8.0 optimization software. The results or output of the proposed model are shown in relation to each decision variable, which represents the number of spectrum packets that have been successfully allocated to each relevant licensee as presented in Table 4. The optimal total efficiency scored when adopting the proposed model is 23.2108.

Table 4

Results of the Proposed Integrated Model

Decision variables	Number of packets
x_{11}	15
x_{12}	14
x_{13}	11
x_{14}	0

All results analyzed are for bandwidths of Type I, where we can conclude that only licensee D is not granted with any spectrum but three licensees, licensees A, B and C are able to obtain their required demands for the spectrum. However, both licensees A and B are granted with their expected demand (i.e., 15 and 14) in which they can operate at their utmost best, while licensee C is granted four more than the minimum required demands (i.e., seven).

CONCLUSION AND FUTURE WORK

We have successfully developed a variant of the integrated model of AHP and IP to assign spectrum volumes to those who truly deserve the spectrum, which is actually based on merits and consideration of the seven criteria. Compared to the existing conventional approaches, the integrated approach is much more efficient. This is true as the approach is able to combine subjective judgments computation of AHP with a mathematically formulated IP to produce a systematic and consistent results. Hence, the strengths of both techniques are captured in this integrated approach. The evaluation of the seven criteria reflects their priorities in term of weights through the employment of the AHP, which needs careful judgments by the regulating body as the authoritative decision maker. The incorporation of priority weights into the IP formulation provides meaningful information for assigning suitable amounts of spectrum to appropriate licensees according to their performance in relation to the seven criteria. Thus, the licensees with high abilities are the ones that will be assigned accordingly by the proposed IP model. This integrated optimization model is able to solve a multi-criteria spectrum assignment problem by way of maximizing the total efficiency, which in turn helps the regulatory body to make beneficial decision making.

Having the integrated model at hand, some analyses can be carried out in the future to see the effects of certain manipulating criteria and priorities. This type of what-if analysis can provide potential results which can be evaluated and thus, aid in meaningful decision-making from various perspectives such as those that relate to profit and cost.

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